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On the Second Law of Thermodynamics

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The second law of thermodynamics has two distinct aspects to its foundations. The first concerns the question of why entropy goes up in the future, and the second, of why it goes down in the past. Statistical physicists tend to be more concerned with the first question and with careful considerations of definition and mathematical detail. The second question is of quite a different nature; it leads into areas of cosmology and quantum gravity, where the mathematical and physical issues are ill understood.

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TWO ASPECTS OF THE SECOND LAW

The very brief remarks that I am making here can in no way reflect the magnitude of the scientific and mathematical indebtedness that I feel towards my brother Oliver Penrose. From a very young age, he provided me both with inspiration and with a depth of realization that in mathematics and in physics there was to be found excitement, mystery, and much logical structure of profound beauty. Though at that early stage in life I had imagined that I would become a doctor, there was always that older brother whom I greatly looked up to, and from whom I gained a profound appreciation that in the "hard sciences" also there was something to strive for that is deeply worthwhile. Only much later, after my early flirtations with pure mathematics, did I finally turn to the study of the physical world in any serious way. The many conversations with Oliver, over a great number of influential years, have helped me to formulate my viewpoints and sharpen my critical sense.

Oliver's own interests took him in the direction of statistical physics,

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where his profound contributions are acknowledged world-wide. Although I had picked up, from Oliver, some of the underlying concepts, statistical physics is an area in which I have never myself felt comfortable; so it was with some considerable awkwardness and uncertainty that I have found myself driven—as a logical consequence of my own interests in the structure of black holes and cosmological space-time singularities—to enter into the discussion of time asymmetry and the origin and nature of the second law of thermodynamics. It seems that coming at this subject as an outsider with an unconventional point of view, I have found myself to be intrigued by issues that appear to be different from those that have traditionally been the concern of most statistical physicists. Sometimes such people would address me with some puzzlement at the fact that my interests in this area would be so much concentrated on matters of cosmology and quantum gravity, while I would seem almost totally to ignore that great body of literature on such matters as Boltzmann's H theorem, microcanonical ensembles, equilibrium fluctuations, and the like. Statistical physicists are themselves not usually particularly interested (professionally) in cosmology —so I might be informed—and certainly not in quantum gravity. Those subjects, it would appear, have almost no relevance to the matters of primary concern to a statistical physicist. On the issues that might more directly *be* their concern, my own silence would, more often than not, merely reflect my own considerable lack of expertise; nevertheless I feel that there is a separate question of relevance here, and it is this that I shall try to address in this short note.

As I see it, there is at least one important difference in motivation between my own interests and those of the main body of statistical physicists. Most conventional physicists are concerned with predicting the future, rather than retrodicting the past. To them, if the second law of thermodynamics holds any mystery—or even if it merely falls marginally short of being rigorously understood—what would need to be understood would be: why it is that the entropy in an isolated system goes *up* in the *future*? To me, the mystery is a different one: why does the entropy go *down* in the *past*?

Superficially, these might seem to be the same question. But on reflection, one sees that the two questions are completely different. We may recall that Boltzmann himself[1] had promoted the idea that the existence of a second law of thermodynamics at our present epoch might be the result of our universe—or at least of our local part of the universe—having encountered an enormous fluctuation of low entropy from which our present increasing entropy would represent a natural proceeding toward thermal equilibrium. On this view, at the times that immediately precede that fluctuation, the time direction of entropy increase would have been opposite to what we experience now. In this picture of things, the entropy of the universe would go up into the past (that is, in the past direction, to the past of this fluctuation) just as it goes up, now, in the time direction that we now refer to as the "future." All the reasoning that leads us to expect that the entropy of our present universe (or universe neighborhood) is increasing into the future, away from this fluctuation, would apply equally well in the past direction, to the past of this fluctuation. In such a scheme, the second law would *not* have held in the past, with respect to the time direction that we use *now*. However, all the cosmological evidence points overwhelmingly to the conclusion that there has been no such turnaround in the direction of the second law—at least so far as our direct (and indirect) observations of the early universe imply. If our second law is the result of an enormous fluctuation, then it must have been a fluctuation of much *greater* enormity and at a much earlier time than Boltzmann could have imagined. Indeed the fluctuation would have to have been so enormous that no explanation of the "anthropic" kind—whereby the very existence of intelligent life provides the reason for the low-entropy "oasis" from which we need to have emerged (cf. ref. 2, p. 354; also ref. 3, p. 589)—comes close to providing an explanation. Thus, the fact that the entropy continues to go *down* in the past, the farther into the past we probe, provides us with a mystery of a quite different order from the more familiar problem of showing why it is that the entropy has a tendency to increase in the future.

These two aspects of the second law, it seems to me, have a quite distinct conceptual status, and the types of mathematical and physical argument that are brought to bear on them appear to be very different. For those arguments that are aimed at showing why the entropy goes up in the future— or, at least, *usually* goes up in the future—one must have a deep understanding of the meaning of the term "entropy" and of the precise mathematical formulations of the various concepts involved. One must also be explicit about the assumptions that need to be invoked in the mathematical derivations. (See Oliver's classic text[4] for a rigorous and comprehensive treatment of these issues.) To ensure that there is a general entropy increase into the future, one must assume the absence of prior correlations in the details of the initial particle motions. It seems "reasonable" to make such an assumption, since within any chosen set of macroscopic parameters, those initial states for which such correlations are essentially absent would form the vast majority. Nevertheless, one must expect that correlations of this nature would be *present*, in *time-reversed* sense, *after* any significant increase in entropy, so one cannot assume, just on general grounds, that the absence of correlations is an all-embracing feature of our inverse. In an important paper, Oliver, together with Ian

Percival,[5] set down an explicit assumption that provides a past/future distinction, which they referred to as the *law of conditional independence*. According to this assumption, the absence of such correlations would be a feature of the remote past, so that deductions concerning the general increase in entropy in the future can then be made (contrast ref. 3, p.589).

My own interest in the second law, on the other hand, has been concerned with the reverse question of why the entropy goes *down* in the *past*. Here, the effect is gross rather than subtle. Delicate issues concerning the definition of entropy, the nature of equilibrium and of fluctuations away from equilibrium, etc.—though they are certainly of great importance when one is concerned with details of the *future* behavior of an isolated system—appear to lie far from the gross type of explanation that would seem to be needed for an understanding of this particular aspect of the second law. My own attempts at an explanation have been described in a series of papers over the years,[6, 3] where I have been promoting a very different kind of criterion for the initial state of the universe from the law of conditional independence of Penrose and Percival, which I have referred to as the *Weyl curvature hypothesis*. It has very little explicit connection with the notion of entropy, as such. Even my earlier attempts at finding a general definition of "gravitational entropy" have had to be abandoned—or at least set aside, for the time being. It is only the Bekenstein–Hawking formula for black-hole entropy which has so far found any specific role to play in my deliberations, namely to provide an estimate for the total entropy of a closed universe. (One finds, by use of this formula, that the entropy in a generic closed universe with baryon number 10^{80} is about 10^{123}, in natural units.)

The role of the Weyl curvature hypothesis is to set up an initial state for the universe which is very special in a particular way—a way that is consistent with what we know about cosmology and astronomy. That the entropy of the early universe is thereby constrained to be extremely low—incomparison with what it would otherwise have been (recall the figure 10^{123})—is, in a sense, incidental to this. But the hypothesis does indeed provide a picuture in which the entropy must go down in the past. On the other hand, it provides no guarantee that the entropy will continue to go up in the future. Perhaps some kind of combination of the law of conditional independence with the Weyl curvature hypothesis would be needed if an understanding of the second law of thermodynamics, in *both* these aspects, is to be obtained (cf. also ref. 3, pp. 633–634).

It may be remarked that, although the Weyl curvature hypothesis may be thought of as a "blunt instrument" in its relation to thermodynamics, it has played a clear-cut role in some precise mathematical theorems of importance to cosmology. In particular, Newman[7] has shown, using a refined form of the Weyl curvature hypothesis due to K. P. Tod—whereby it is assumed that the initial big-bang singularity is conformally regular as a hypersurface—that one can deduce that, with appropriate equations of state, the early universe must be of Friedmann–Robertson–Walker type (as, indeed, it is observed closely to be).

The other main role of the Weyl curvature hypothesis lies in highly speculative matters to do with quantum gravity and with the measurement problem in quantum mechanics. My own interests have taken me in the direction of these particular questions,[8] but it will be a long while before the mathematical status of any of these considerations comes close to that enjoyed within the subject of statistical physics.

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